Electric Current and Current Density; Resistance and Resistivity; Ohm’s Law; Power in Electric Circuits; Semiconductors and Superconductors; Work; Energy and EMF; Resistances in Series and Parallel;

Current and resistance

Electric Current

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current.

If there is to be an electric current through a given surface, there must be a net flow of charge through that surface.

Two examples clarify our meaning.

1. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 106 m/s.

   If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second -but there is no net transport of charge and thus no current through the wire.

However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a net transport of charge and thus an electric current through the wire.

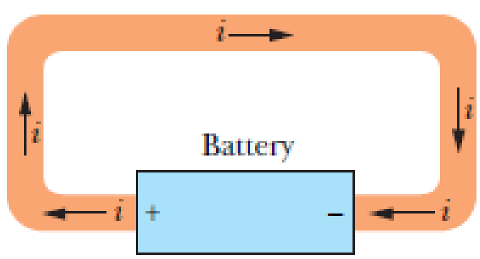
2. The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million

coulombs per second. There is no net transport of charge, however, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

Fig. 1-a, any isolated conducting loop - regardless of whether it has an excess charge - is all at the same potential. No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.

(a)

If, as in Fig. 1-b, we insert a battery in the loop, the conducting loop is no longer at a single potential. Electric fields act inside the material making up the loop, exerting forces on the conduction electrons, causing them to move and thus establishing a current. After a very short time, the electron flow reaches a constant value and the current is in its steady state (it does not vary with time).



(b)

Figure 2 shows a section of a conductor, part of a conducting loop in which current has been established. If charge dq passes through a hypothetical plane (such as **aa′**) in time *dt*, then the current *i* through that plane is defined as

$$i = \frac{dq}{dt}$$

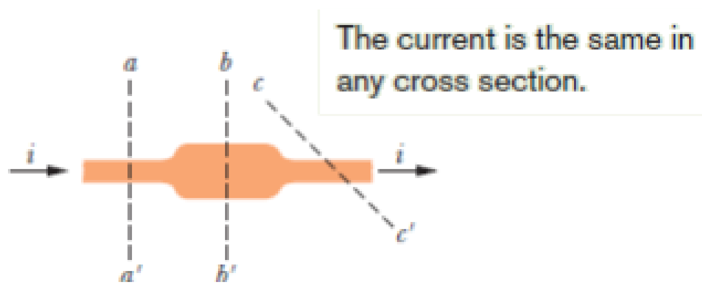


Fig. 2 The current *i* through the conductor has the same value at planes *aa′*, *bb′*, and *cc′*.

We can find the charge that passes through the plane in a time interval extending from *0* to *t* by integration:

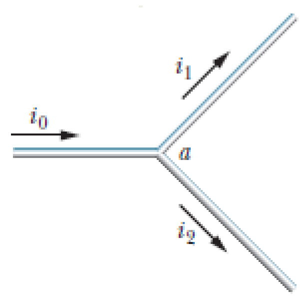
$$q = \int dq = \int_0^t i\, dt,$$

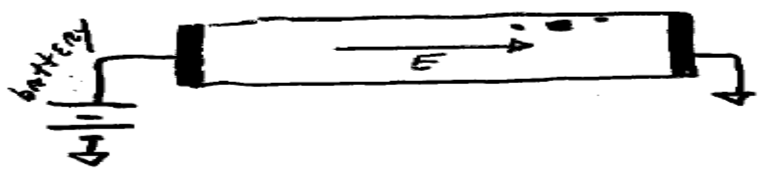
Under steady-state conditions, the current is the same for planes aa′, bb′, and cc′ and indeed for all planes that pass completely through the conductor, no matter what their location or orientation. This follows from the fact that charge is conserved.

1 ampere = 1 A = 1 coulomb per second = 1 C/s.

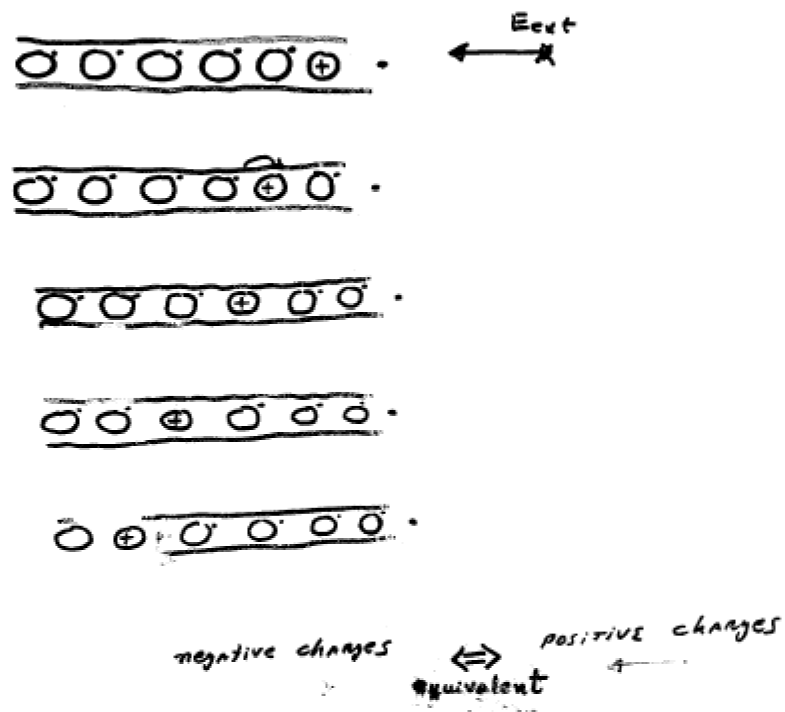
Current is a scalar because both charge and time in that equation are scalars. Figure shows a conductor with current i_o splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$i_0 = i_1 + i_2.$$





Electric fields $\vec{E}$ act inside the material, exerting forces on the conduction electrons, conduction electrons, thus establishing a current



Eext
negative charges
⇔
equivalent
positive charges

Current Density

Current density has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the element as *J. dA* where *dA* is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}.$$

If the current is uniform across the surface and parallel to *dA,* then *J* is also uniform and parallel to *dA*

$$i = \int J\, dA = J \int dA = JA$$

$$J = \frac{i}{A}$$

SI unit for current density is the ampere per square meter (A/m2).

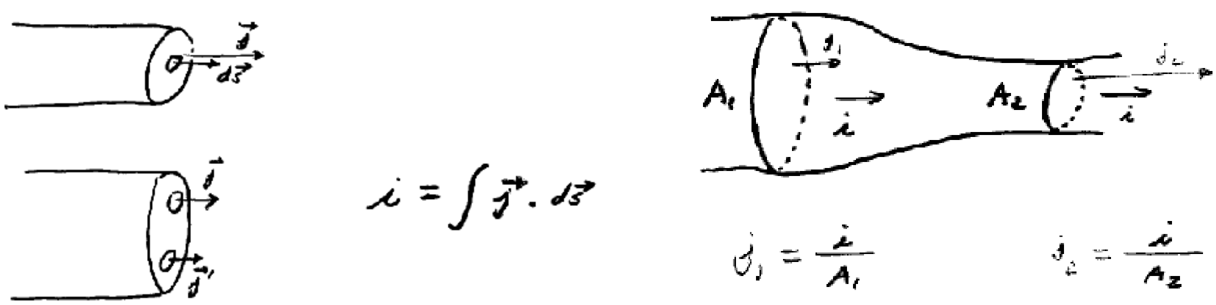


Figure below shows how current density can be represented with a similar set of lines, which we can call streamlines.

The current, which is toward the right in figures, makes a transition from the wider conductor at the left to the narrower conductor at the right. Because charge is conserved during the transition, the amount of charge and thus the amount of current cannot change.

However, the current density does change - it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.

Figure

Drift Speed

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction.

When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to drift with a drift speed $\mathbf{v_d}$ in the direction opposite that of the applied electric field that causes the current.

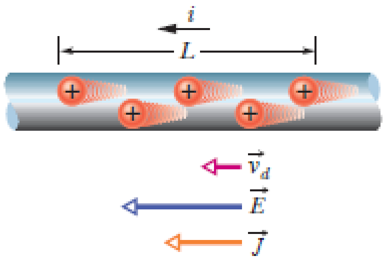
The drift speed is tiny compared with the speeds in the random motion.

For example, in the copper conductors of household wiring, electron drift speeds are perhaps 10^{-5} or 10^{-4} m/s, whereas the random-motion speeds are around 10^{6} m/s.

To relate the drift speed $\mathbf{v_d}$ of the conduction electrons in a current through a wire to the magnitude *J* of the current density in the wire. For convenience, Fig. below shows the equivalent drift of positive charge carriers in the direction of the applied electric field.

Let us assume that these charge carriers all move with the same drift speed vd and that the current density J is uniform across the wire's cross-sectional area A.

The number of charge carriers in a length L of the wire is nAL, where n is the number of carriers per unit volume. The total charge of the carriers in the length L, each with charge e, is then



$$q = (nAL)e.$$

Because the carriers all move along the wire with speed v_d, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Equation 26-1 tells us that the current i is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d.$$

Solving for v_d and recalling Eq. 26-5 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

or, extended to vector form,

$$\vec{J} = (ne)\vec{v}_d.$$

Resistance and Resistivity

The resistance between any two points of a conductor can be determine by applying a potential difference *V* between those points and measuring the current *i* that results. The resistance *R* is then

$$R = \frac{V}{i} \qquad 1$$

The SI unit for resistance is volt per ampere. We give it a special name, the ohm (symbol ( ); that is, 1 ohm = 1 Ω = 1 volt per ampere

If we write $$i = \frac{V}{R} \qquad 2$$

We see that, for a given V, the greater the resistance, the smaller the current.

As we have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference V across a particular resistor but on the electric field *E* at a point in a resistive material. Instead of dealing with the current *i* through the resistor, we deal with the current density *J* at the point in question.

Instead of the resistance R of an object, we deal with the resistivity of the material

$$\rho = \frac{E}{J} \qquad 3$$

If we combine the SI units of E and J according to above Eq., we get, for the unit of resisitivity, the ohm-meter:

$$\frac{\text{unit } (E)}{\text{unit } (J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\text{m} = \Omega\cdot\text{m}.$$

$$\vec{E} = \rho\vec{J}. \qquad 4$$

Equations 3 and 4 hold only for isotropic materials - materials whose electrical properties are the same in all directions

We often speak of the conductivity s of a material. This is simply the reciprocal of its resistivity, so

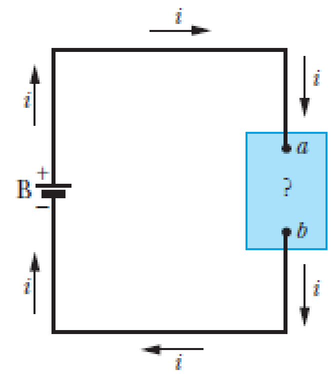
$$\sigma = \frac{1}{\rho}$$

$$\vec{J} = \sigma\vec{E}.$$

Power in Electric Circuits

Figure shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance.

The battery maintains a potential difference of magnitude V across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b.



Potential differences set up by the battery are maintained, a steady current *i* is produced in the circuit, directed from terminal *a* to terminal *b*. The amount of charge dq that moves between those terminals in time interval *dt* is equal to *i dt*.

This charge *dq* moves through a decrease in potential of magnitude *V*, and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\ V = i\ dt\ V.$$

The power *P* associated with that transfer is the rate of transfer ***dU/dt***,

$$P = iV$$

For a resistor or some other device with resistance *R*,

$$P = i^2 R$$

$$P = \frac{V^2}{R}$$